Why MB/UE physics prefer LO PDFs

Torbjörn Sjöstrand
Dept. of Astronomy and Theoretical Physics, Lund University

Original date: 24 Aug 2012
Last changed: 24 Aug 2014

Recently tunes with NLO PDFs have appeared for Pythia 8. The following very brief note explains why usage of them is highly discouraged.

One key feature of MB (minimum bias) and UE (underlying event) studies is that much of the physics is played out at low transverse momenta, say $p_\perp$ around 2 GeV. Therefore PDFs at very small x scales are probed, down to around 10^{-8}, for Q scales that may go below 1 GeV. It is the behaviour in this region that we want to discuss next, *not* the values relevant for Higgs production, say.

For an up-to-date and comprehensive overview of PDFs we refer to [1].

1 The small-x behaviour in LO

In a LO framework the PDFs have a clear physical interpretation as the number density of partons. This means that its behaviour can be related directly to measurable quantities. More specifically there are two main measurements that lead credence to a small-x behaviour where $xf_i(x, Q^2)$ is constant or even slowly rising for $x \to 0$ at a fixed Q^2 around $1 - 4$ GeV^2, for gluons and sea quarks.

The first is F_2 measurements at HERA, which displays the above behaviour. In LO the F_2 is related to the valence and sea quark distributions, but at small x the latter are driven by the gluon, at comparable x values, which therefore should have a similar shape.

The second is the rise of the pp/$\overline{\mathrm{p}}$p cross section with energy. This can be directly related to the rise of $xg(x, Q^2_{\mathrm{ref}})$ with decreasing x for a fixed small reference scale. That is, to some approximation we expect $\sigma_{\mathrm{pp}}(s) \propto xg(x, Q^2_{\mathrm{ref}})$ for $x \propto 1/s \to 0$, the so-called Regge–Gribov limit. Also for models that describe the rise of $\sigma_{\mathrm{pp}}(s)$ in terms of eikonalized minijet cross sections (related to MPI ideas) the small-x behaviour of the gluon plays a similar role.

Therefore, while not completely pinned down, the LO description has some trustworthiness for the form of the PDFs for small x, from direct measurements of physical quantities.

2 The problem with NLO

At NLO, the PDFs no longer have a probabilistic interpretation, and their behaviour is less directly related to physical quantities. PDFs need not even be positive definite, as first introduced in [2] to improve agreement with data.

So indeed, for many MRST/MSTW tunes, the gluon is negative at small x for the low Q_0 starting scale at around 1 – 2 GeV. In CTEQ fits the parametrized form does not allow the gluon PDF to turn negative, but it is very close to zero at small x and Q. One reason CTEQ gets away with this is that only data above $Q^2 = 4$ GeV^2 are used, while MRST/MSTW go down to 2 GeV^2.

The key constraint on the low-x gluon PDF comes from the DIS F_2, where $\mathrm{d}F_2/\mathrm{d}\ln Q^2$ is driven by $g \to q\bar{q}$ branchings. At LO the $P_{q/g}(z)$ splitting kernel is quite flat, so the x of the measured quark is closely correlated with that of the mother gluon. At NLO $P_{q/g}(z) \propto 1/z$ for small z, and the integral over z values introduces an approximate $\ln(1/x)$ factor. Since the gluon is now probed more non-locally, the $\mathrm{d}F_2/\mathrm{d}\ln Q^2$ at small x would become too big if not the positive contribution from medium-to-high-x gluons (derived from $\mathrm{d}F_2/\mathrm{d}\ln Q^2$ in that region, and from other measurements) were combined with a negative contribution from low-x gluons.

The problem remains in NNLO, and is even aggravated by more singular splitting kernels. Attempts at an all-order resummation of $\ln(1/x)$ terms gives a gluon that is more like LO than like NLO. For details see section 4.3 in [1].

The problem becomes less relevant for higher-$p_\perp$ processes, because

- DGLAP evolution fills up the lower-x region,
- kinematics is restricted to higher x vales, and
- α_s is reduced.

In summary, NLO implies small-x corrections proportional to $\ln(1/x)$, that may drive PDFs negative at small x and Q.

3 A toy NLO calculation

To illustrate this, consider a process in pp collision, as a convolution of a ME and two PDFs. For simplicity, study only the interplay between the ME and the PDF on one side of the event, given the x scale there. A generalization to one x scale on each side of the event is straightforward.

By standard perturbation theory the effect of typical NLO matrix elements in pp collisions leads to an enhancement by a factor

$$\frac{\mathrm{ME_{NLO}}}{\mathrm{ME_{LO}}} = 1 + \alpha_\mathrm{s}(A_1 \ln(1/x) + A_0) \tag{1}$$

The divergent $\ln(1/x)$ behaviour above is largely to be compensated in the definition of NLO PDFs. With

$$\frac{\mathrm{PDF_{NLO}}}{\mathrm{PDF_{LO}}} = 1 + \alpha_\mathrm{s}(B_1 \ln(1/x) + B_0) \tag{2}$$

it should follow that $B_1 \approx -A_1$. Thereby the product of ME times PDF is well-behaved to $\mathcal{O}(\alpha_\mathrm{s})$. There is a cross-term of $\mathcal{O}(\alpha_\mathrm{s}^2)$, which is beyond the stated NLO accuracy.

We now see the numerical problem. For reasonably large x and Q^2 scales, where $\alpha_\mathrm{s}(Q^2)$ is small, say $\alpha_\mathrm{s} A_1 \ln(1/x) = 0.2$, the logarithmic terms give

$$\frac{\mathrm{ME_{NLO}}\,\mathrm{PDF_{NLO}}}{\mathrm{ME_{LO}}\,\mathrm{PDF_{LO}}} = (1 + 0.2)(1 - 0.2) = 0.96\ , \tag{3}$$

i.e. they cancel to a good approximation. But if instead x and Q^2 are small, say $\alpha_{\mathrm{s}} A_1 \ln(1/x) = 2$, then

$$\frac{\mathrm{ME}_{\mathrm{NLO}}\,\mathrm{PDF}_{\mathrm{NLO}}}{\mathrm{ME}_{\mathrm{LO}}\,\mathrm{PDF}_{\mathrm{LO}}} = (1+2)(1-2) = -3, \tag{4}$$

i.e. the PDF becomes negative, the cross-term of $\mathcal{O}(\alpha_{\mathrm{s}}^2)$ dominates, and the simple calculation derails.

4 Phenomenology in PYTHIA 8

Tunes have been produced both with LO and with NLO PDFs. In general they both give comparably good descriptions of data, which would seem to contradict the arguments above.

What is notable is that tunes for NLO PDFs require a significantly smaller $p_{\perp 0}$ scale, where $p_{\perp 0}$ is used to tame the $1/p_{\perp}^4$ divergence of the QCD cross sections to $1/(p_{\perp}^2 + p_{\perp 0}^2)^2$. This reduced $p_{\perp 0}$ is precisely what is needed to compensate for the low amount of small-x gluons in NLO PDFs. It is here useful to recall that, for the integrated QCD cross sections, it is the *number* density $f_i(x, Q^2)$ that enters the integrals, rather than the momentum-weighted $x f_i(x, Q^2)$ expressions. Thus the small-x partons play an important role.

In the NLO tunes, the MPI collisions would tend to be symmetric, i.e. with $x_1 \sim x_2$, and both not too small. Asymmetric collisions, where one x is small, would be killed by the respective NLO PDFs vanishing or at least being tiny there (a negative PDF is reset to 0 in PYTHIA). One therefore expects to find differences in the rapidity spectrum of minijets from MPIs. The main reason that MPIs contribute so significantly to the charged multiplicity distribution and to $\mathrm{d}n_{\mathrm{chg}}/\mathrm{d}\eta$ is not the minijets in itself, however, but the strings that are stretched out to the beam remnants. (Or, with colour reconnection included, between the different MPIs.) Therefore the number of MPIs may be more important than their exact location in rapidity.

The bottom line is that the MPI and string fragmentation frameworks are sufficiently resilient that a rather significant change of PDF shape can be compensated by a retuning of relevant parameters. Differences could probably be found in more detailed studies, e.g. in $\mathrm{d}n_{\mathrm{minijet}}/\mathrm{d}\eta$ distributions over a large η range. Irrespective of that, there is no reason to use NLO PDFs in regions where they are known not to be trustworthy.

5 Recommendation

If one is not satisfied to use an LO PDF set throughout, PYTHIA 8 offers the possibility to use two separate PDF sets in the simulation, with the switch `PDF:useHard = on`.

One set can then be used exclusively for the hard process itself, where presumably both x and Q^2 are large. None of the issues raised above therefore matter, and one is at liberty to use LO or NLO PDFs to calculate the (differential and total) cross section of the process. Insofar as the PDFs are combined with the built-in LO MEs, the overall formal accuracy would still be LO, but numerically NLO PDFs could still end up closer to known fully-NLO results.

The other set would be used for MPIs and for ISR. In both of these components there can be quite hard scales, but the bulk of activity in them is obtained at small $p_\perp$ scales, where it is important to handle the small-x issues raised above. Therefore a LO PDF is strongly recommended for this application. We also recall that ISR generated with the standard backwards evolution scheme [3] is based on *ratios* of PDFs. Therefore many of the differences between PDF sets divide out, notably away from the low-x region.

An additional advantage of a two-PDF setup is that it becomes possible to explore a range of PDFs for the hard process without any necessity to redo the UE(/MB) tune.

Acknowledgements

Input, especially from Robert Thorne, is gratefully acknowledged. The author should solely bear the blame for opinions, simplifications and errors in this note, however.

References

[1] A. De Roeck and R. S. Thorne, Prog. Part. Nucl. Phys. **66** (2011) 727 [arXiv:1103.0555 [hep-ph]].

[2] A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. C **23** (2002) 73 [hep-ph/0110215].

[3] T. Sjöstrand, Phys. Lett. B **157** (1985) 321.